**Investments Answers to Problem Sheet 8 Lent Term 2024**

1. A 5 year bond with a coupon of 7%, that pays interest annually, has its next coupon due in 12 months. It yields 5% per annum.

(a) compute the price of the bond.

(b) compute the duration of the bond. Use this to compute approximately how the price of the bond will change if yields rise to 6%.

(c) Compute the convexity of the bond. Use this to obtain a more precise estimate of the change in the price which will occur if the yield rises to 6%.

**(a) The price of the bond is 7/1.05 + 7/1.052 + 7/1.053+ 7/1.054 + 107/1.055 = 108.66**

**(b) The duration of the bond is (7\*1/1.05 + 7\*2/1.052 + 7\*3/1.053+ 7\*4/1.054 + 107\*5/1.055)/108.66 = 4.41 years. The modified duration is 4.41/1.05 = 4.20. A 1% rise in yield can be expected to reduce the price by 4.2%, from £108.66 to £104.09.**

**(c) The convexity of the bond is (7\*1\*2/1.05 + 7\*2\*3/1.052 + 7\*3\*4/1.053+ 7\*4\*5/1.054 + 107\*5\*6/1.055)/(108.66\*1.052) = 22.99 years2. This implies a further price correction from a 1% shift of +22.99\*1%2/2 or +.1% to £104.22.**

2. Firm XYZ is required to make a $5M payment in 1 year and a $4M payment in 3 years. The yield curve is flat at 10% APR with semi-annual compounding. Firm XYZ wants to form a portfolio using 1-year and 4-year U.S. strips to fund the payments. How much of each strip must the portfolio contain for it to still be able to fund the payments after a shift in the yield curve?

The value of the liabilities is given by:

5M/[1+0.1/2]2 + 4M/[1+0.1/2]6 = 4.5351M + 2.9849M = 7.5200M

The duration of the liabilities is given by:

1\* [4.5351/7.5200] + 3\* [2.9849/7.5200] = 1.7938 years.

Modified Duration = 1.7938 years/(1+0.10/2) = 1.708 years.

Let A1 be the portfolio s dollar investment in the 1-year strips and A4 be the portfolio s dollar investment in the 4-year strips.

The dollar value of the portfolio must equal the value of the liabilities. So

A1 + A4= 7.5200M.

The duration of the portfolio equals: w1D1 +(1- w1) D4

where w1 = A1/ 7.5200M. The (Modified) duration of the 1-year strip is 1/(1+0.1/2) and the duration of the 4-year strip is 4/(1+0.1/2).

Setting the duration of the portfolio equal to the duration of the liabilities gives:

1.708 = w1 D1 + (1- w1) D4 = w1 (.95) + (1- w1) 3.80 => w1 = 0.732.

Thus,

A1 = 0 .732 ´ 7.5200M = 5.50M

A4 = 7.5200M - 5.502M = 1.99M.

3. You are managing a portfolio of $1 million, Your target duration is 10 years, and you can choose from two bonds: a zero-coupon bond with maturity of 5 years, and a perpetuity, each currently yielding 5%

1. How much of each bond will you hold in your portfolio?
2. How will these fractions change next year if target duration is now 9 years?

**Answer:**

The duration of the perpetuity is: 1.05/0.05 = 21 years

i)Call w the weight of the zero-coupon bond. Then:

(w × 5) + [(1 – w) × 21] = 10  w = 11/16 = 0.6875

Therefore, the portfolio weights would be as follows: 11/16 invested in the zero and 5/16 in the perpetuity.

ii)Next year, the zero-coupon bond will have a duration of 4 years and the perpetuity will still have a 21-year duration. To obtain the target duration of nine years, which is now the duration of the obligation, we again solve for w:

(w × 4) + [(1 – w) × 21] = 9  w = 12/17 = 0.7059

So, the proportion of the portfolio invested in the zero increases to 12/17 and the proportion invested in the perpetuity falls to 5/17.

1. You will be paying $10,000 a year in tuition expenses at the end of the next 2 years. Bonds currently yield 8%.
2. What is the present value and duration of your obligation?

1. What maturity zero-coupon bond would immunize your obligation?
2. Suppose you buy a zero-coupon bond with value and duration equal to your obligation. Now suppose that rates immediately increase to 9%. What happens to your net position? What if the rates fall to 7%?

**Answer:**

1. PV of the obligation = $10,000 × Annuity factor (8%, 2) = $17,832.65

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| --- | --- | --- | --- | --- |
| Time until Payment (years) | Cash Flow | PV of CF (Discount rate = 8%) | Weight | Column (1) × Column (4) |
| 1 | $10,000.00 | $9,259.259 | 0.51923 | 0.51923 |
| 2 | $10,000.00 | $8,573.388 | 0.48077 | 0.96154 |
| Column Sums | | $17,832.647 | 1.00000 | 1.48077 |

Duration = 1.4808 years

1. A zero-coupon bond maturing in 1.4808 years would immunize the obligation. Since the present value of the zero-coupon bond must be $17,832.65, the face value (i.e., the future redemption value) must be:

$17,832.65 × 1.081.4808 = $19,985.26

1. If the interest rate increases to 9%, the zero-coupon bond would decrease in value to:



The present value of the tuition obligation would decrease to: $17,591.11. The net position decreases in value by: $0.19

If the interest rate decreases to 7%, the zero-coupon bond would increase in value to:



The present value of the tuition obligation would increase to: $18,080.18. The net position decreases in value by: $0.19.

The reason the net position changes at all is that, as the interest rate changes, so does the duration of the stream of tuition payments.

1. Long-term Treasury bonds currently are selling at yields to maturity of nearly 8%. You expect interest rates to fall. The rest of the market thinks that they will remain unchanged over the coming year. In each question, choose the bonds that will provide the higher holding-period return over the next year if you are correct. Briefly explain your answer.
2. i. A Baa-rated bond with coupon rate 8% and time to maturity 20 years.

ii. An Aaa-rated bond with coupon rate of 8% and time to maturity 20 years.

1. i. An A-rated bond with coupon rate 4% and maturity 20 years.

ii.An A-rated bond with coupon rate 8% and maturity 20 years.

1. i. A 6% coupon T-bond with maturity 20 years and YTM = 8%.

ii. A 9% coupon T-bond with maturity 20 years and YTM = 8%.

**Answer:**

In each case, choose the longer-duration bond in order to benefit from a rate decrease.

a. ii. The Aaa-rated bond has the lower yield to maturity and therefore the longer duration.

b. i. The lower-coupon bond has the longer duration

c. i. The lower coupon bond has the longer duration.